RECOGNISING ACHIEVEMENT

ADVANCED GCE **4756/01**

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

THURSDAY 15 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
- Answer Booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has cartesian equation $(x^2 + y^2)^2 = 3xy^2$.

(i) Show that the polar equation of the curve is $r = 3\cos\theta\sin^2\theta$. [3]

(ii) Hence sketch the curve. [3]

(b) Find the exact value of $\displaystyle\int_0^1 \frac{1}{\sqrt{4 - 3x^2}}\,dx$. [5]

(c) **(i)** Write down the series for $\ln(1 + x)$ and the series for $\ln(1 - x)$, both as far as the term in x^5. [2]

(ii) Hence find the first three non-zero terms in the series for $\ln\left(\dfrac{1+x}{1-x}\right)$. [2]

(iii) Use the series in part **(ii)** to show that $\displaystyle\sum_{r=0}^{\infty} \frac{1}{(2r+1)4^r} = \ln 3$. [3]

2 You are given the complex numbers $z = \sqrt{32}(1 + \mathrm{j})$ and $w = 8\left(\cos\frac{7}{12}\pi + \mathrm{j}\sin\frac{7}{12}\pi\right)$.

(i) Find the modulus and argument of each of the complex numbers z, z^*, zw and $\dfrac{z}{w}$. [7]

(ii) Express $\dfrac{z}{w}$ in the form $a + b\mathrm{j}$, giving the exact values of a and b. [2]

(iii) Find the cube roots of z, in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [4]

(iv) Show that the cube roots of z can be written as

$$k_1 w^*, \quad k_2 z^* \quad \text{and} \quad k_3 \mathrm{j} w,$$

where k_1, k_2 and k_3 are real numbers. State the values of k_1, k_2 and k_3. [5]

3 **(i)** Given the matrix $\mathbf{Q} = \begin{pmatrix} 2 & -1 & k \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ (where $k \neq 3$), find $\mathbf{Q}^{-1}$ in terms of k.

Show that, when $k = 4$, $\mathbf{Q}^{-1} = \begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$. [6]

The matrix $\mathbf{M}$ has eigenvectors $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$, with corresponding eigenvalues 1, −1 and 3 respectively.

(ii) Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$, and hence find the matrix $\mathbf{M}$. [7]

(iii) Write down the characteristic equation for $\mathbf{M}$, and use the Cayley-Hamilton theorem to find integers a, b and c such that $\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. [5]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1.$$ [3]

(ii) Solve the equation $4\cosh^2 x + 9\sinh x = 13$, giving the answers in exact logarithmic form. [6]

(iii) Show that there is only one stationary point on the curve

$$y = 4\cosh^2 x + 9\sinh x,$$

and find the y-coordinate of the stationary point. [4]

(iv) Show that $\int_0^{\ln 2} (4\cosh^2 x + 9\sinh x)\,dx = 2\ln 2 + \frac{33}{8}$. [5]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations $x = \lambda \cos\theta - \frac{1}{\lambda}\sin\theta$, $y = \cos\theta + \sin\theta$, where λ is a positive constant.

(i) Use your calculator to obtain a sketch of the curve in each of the cases

$$\lambda = 0.5, \quad \lambda = 3 \quad \text{and} \quad \lambda = 5.$$ [3]

(ii) Given that the curve is a conic, name the type of conic. [1]

(iii) Show that y has a maximum value of $\sqrt{2}$ when $\theta = \frac{1}{4}\pi$. [2]

(iv) Show that $x^2 + y^2 = (1 + \lambda^2) + \left(\frac{1}{\lambda^2} - \lambda^2\right)\sin^2\theta$, and deduce that the distance from the origin of any point on the curve is between $\sqrt{1 + \frac{1}{\lambda^2}}$ and $\sqrt{1 + \lambda^2}$. [6]

(v) For the case $\lambda = 1$, show that the curve is a circle, and find its radius. [2]

(vi) For the case $\lambda = 2$, draw a sketch of the curve, and label the points A, B, C, D, E, F, G, H on the curve corresponding to $\theta = 0, \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi, \pi, \frac{5}{4}\pi, \frac{3}{2}\pi, \frac{7}{4}\pi$ respectively. You should make clear what is special about each of these points. [4]

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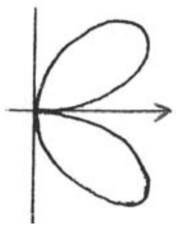
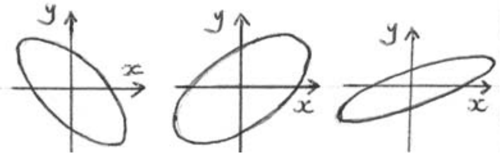
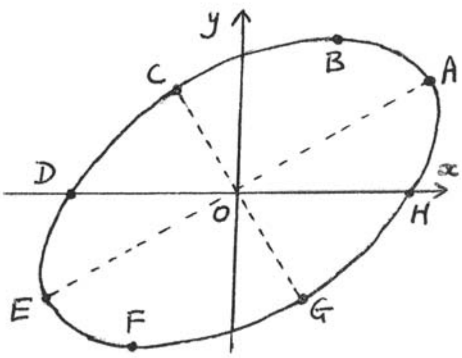
1(a)(i)	$x = r\cos\theta,\ \ y = r\sin\theta$ $(r^2\cos^2\theta + r^2\sin^2\theta)^2 = 3(r\cos\theta)(r\sin\theta)^2$ $r^4 = 3r^3\cos\theta\sin^2\theta$ $r = 3\cos\theta\sin^2\theta$	M1 A1 A1 ag **3**	(M0 for $x = \cos\theta,\ \ y = \sin\theta$)
(ii)		B1 B1 B1 **3**	Loop in 1st quadrant Loop in 4th quadrant Fully correct curve *Curve may be drawn using continuous or broken lines in any combination*
(b)	$\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,\mathrm{d}x = \left[\frac{1}{\sqrt{3}}\arcsin\frac{\sqrt{3}\,x}{2}\right]_0^1$ $= \frac{1}{\sqrt{3}}\arcsin\frac{\sqrt{3}}{2}$ $= \frac{\pi}{3\sqrt{3}}$	M1 A1A1 M1 A1 **5**	For arcsin For $\frac{1}{\sqrt{3}}$ and $\frac{\sqrt{3}\,x}{2}$ Exact numerical value *Dependent on first M1* (M1A0 for $60/\sqrt{3}$)
	OR — M1 Put $\sqrt{3}\,x = 2\sin\theta$ — A1 $\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \frac{1}{\sqrt{3}}\,\mathrm{d}\theta$ — A1 $= \frac{\pi}{3\sqrt{3}}$ — M1A1		Any sine substitution For $\int \frac{1}{\sqrt{3}}\,\mathrm{d}\theta$ *M1 dependent on first M1*
(c)(i)	$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \ldots$ $\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \ldots$	B1 B1 **2**	*Accept unsimplified forms*
(ii)	$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$ $= 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \ldots$	M1 A1 **2**	 Obtained from two correct series *Terms need not be added* If M0, then B1 for $2x + \frac{2}{3}x^3 + \frac{2}{5}x^5$

(iii)	$\sum_{r=0}^{\infty}\frac{1}{(2r+1)4^r}=1+\frac{1}{3\times4}+\frac{1}{5\times4^2}+\ldots$ $=2\times\frac{1}{2}+\frac{2}{3}\times(\frac{1}{2})^3+\frac{2}{5}\times(\frac{1}{2})^5+\ldots$ $=\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)=\ln 3$	B1 B1 B1 ag **3**	*Terms need not be added* For $x=\frac{1}{2}$ seen or implied Satisfactory completion
2 (i)	$\lvert z\rvert=8,\quad \arg z=\frac{1}{4}\pi$ $\lvert z^*\rvert=8,\quad \arg z^*=-\frac{1}{4}\pi$ $\lvert zw\rvert=8\times8=64$ $\arg(zw)=\frac{1}{4}\pi+\frac{7}{12}\pi=\frac{5}{6}\pi$ $\left\lvert\frac{z}{w}\right\rvert=\frac{8}{8}=1$ $\arg(\frac{z}{w})=\frac{1}{4}\pi-\frac{7}{12}\pi=-\frac{1}{3}\pi$	B1B1 B1 ft B1 ft B1 ft B1 ft B1 ft **7**	*Must be given separately Remainder may be given in exponential or $r\,\text{cjs}\,\theta$ form* (B0 for $\frac{7}{4}\pi$) (B0 if left as $8/8$)
(ii)	$\frac{z}{w}=\cos(-\frac{1}{3}\pi)+\mathrm{j}\sin(-\frac{1}{3}\pi)$ $=\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{j}$ $a=\frac{1}{2},\quad b=-\frac{1}{2}\sqrt{3}$	M1 A1 **2**	If M0, then B1B1 for $\frac{1}{2}$ and $-\frac{\sqrt{3}}{2}$
(iii)	$r=\sqrt[3]{8}=2$ $\theta=\frac{1}{12}\pi$ $\theta=\frac{\pi}{12}+\frac{2k\pi}{3}$ $\theta=-\frac{7}{12}\pi,\quad \frac{3}{4}\pi$	B1 ft B1 M1 A1 **4**	Accept $\sqrt[3]{8}$ Implied by one further correct (ft) value *Ignore values outside the required range*
(iv)	$w^*=8\mathrm{e}^{-\frac{7}{12}\pi\mathrm{j}}$, so $2\mathrm{e}^{-\frac{7}{12}\pi\mathrm{j}}=\frac{1}{4}w^*$ $k_1=\frac{1}{4}$ $z^*=8\mathrm{e}^{-\frac{1}{4}\pi\mathrm{j}}=-8\mathrm{e}^{\frac{3}{4}\pi\mathrm{j}}$ So $2\mathrm{e}^{\frac{3}{4}\pi\mathrm{j}}=-\frac{1}{4}z^*$ $k_2=-\frac{1}{4}$ $\mathrm{j}w=8\mathrm{e}^{(\frac{1}{2}\pi+\frac{7}{12}\pi)\mathrm{j}}=8\mathrm{e}^{\frac{13}{12}\pi\mathrm{j}}$ $=-8\mathrm{e}^{\frac{1}{12}\pi\mathrm{j}}$, so $2\mathrm{e}^{\frac{1}{12}\pi\mathrm{j}}=-\frac{1}{4}\mathrm{j}w$ $k_3=-\frac{1}{4}$	B1 ft M1 A1 ft M1 A1 ft **5**	Matching *w** to a cube root with argument $-\frac{7}{12}\pi$ and $k_1=\frac{1}{4}$ or ft ft is $\frac{r}{8}$ Matching *z** to a cube root with argument $\frac{3}{4}\pi$ *May be implied* ft is $-\frac{r}{\lvert z^*\rvert}$ Matching j*w* to a cube root with argument $\frac{1}{12}\pi$ *May be implied* OR M1 for $\arg(\mathrm{j}w)=\frac{1}{2}\pi+\arg w$ *(implied by $\frac{13}{12}\pi$ or $-\frac{11}{12}\pi$)* ft is $-\frac{r}{8}$

3 (i)	$\mathbf{Q}^{-1}=\frac{1}{k-3}\begin{pmatrix}-1 & k+2 & -1\\ 1 & 4-3k & k-2\\ 1 & -5 & 1\end{pmatrix}$ When $k=4$, $\mathbf{Q}^{-1}=\begin{pmatrix}-1 & 6 & -1\\ 1 & -8 & 2\\ 1 & -5 & 1\end{pmatrix}$	M1 A1 M1 A1 M1 A1 **6**	Evaluation of determinant *(must involve k)* For $(k-3)$ Finding at least four cofactors *(including one involving k)* Six signed cofactors correct *(including one involving k)* Transposing and dividing by det *Dependent on previous M1M1* $\mathbf{Q}^{-1}$ correct (in terms of *k*) and result for $k=4$ stated After 0, SC1 for $\mathbf{Q}^{-1}$ when $k=4$ obtained correctly with some working
(ii)	$\mathbf{P}=\begin{pmatrix}2 & -1 & 4\\ 1 & 0 & 1\\ 3 & 1 & 2\end{pmatrix}$, $\mathbf{D}=\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 3\end{pmatrix}$ $\mathbf{M}=\mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ $=\begin{pmatrix}2 & -1 & 4\\ 1 & 0 & 1\\ 3 & 1 & 2\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 3\end{pmatrix}\begin{pmatrix}-1 & 6 & -1\\ 1 & -8 & 2\\ 1 & -5 & 1\end{pmatrix}$ $=\begin{pmatrix}2 & 1 & 12\\ 1 & 0 & 3\\ 3 & -1 & 6\end{pmatrix}\begin{pmatrix}-1 & 6 & -1\\ 1 & -8 & 2\\ 1 & -5 & 1\end{pmatrix}$ $=\begin{pmatrix}11 & -56 & 12\\ 2 & -9 & 2\\ 2 & -4 & 1\end{pmatrix}$	B1B1 B2 M1 A2 **7**	For B2, order must be consistent Give B1 for $\mathbf{M}=\mathbf{P}^{-1}\mathbf{D}\mathbf{P}$ or $\begin{pmatrix}2 & -1 & 4\\ 1 & 0 & 1\\ 3 & 1 & 2\end{pmatrix}\begin{pmatrix}-1 & 6 & -1\\ -1 & 8 & -2\\ 3 & -15 & 3\end{pmatrix}$ Good attempt at multiplying two matrices (no more than 3 errors), leaving third matrix in correct position Give A1 for five elements correct Correct **M** implies B2M1A2 5-8 elements correct implies B2M1A1
(iii)	Characteristic equation is $(\lambda-1)(\lambda+1)(\lambda-3)=0$ $\lambda^3-3\lambda^2-\lambda+3=0$ $\mathbf{M}^3=3\mathbf{M}^2+\mathbf{M}-3\mathbf{I}$ $\mathbf{M}^4=3\mathbf{M}^3+\mathbf{M}^2-3\mathbf{M}$ $=3(3\mathbf{M}^2+\mathbf{M}-3\mathbf{I})+\mathbf{M}^2-3\mathbf{M}$ $=10\mathbf{M}^2-9\mathbf{I}$ $a=10,\ b=0,\ c=-9$	B1 M1 A1 M1 A1 **5**	In any correct form *(Condone omission of =0)* **M** satisfies the characteristic equation Correct expanded form *(Condone omission of* **I** *)*

4 (i)	$\cosh^2 x = [\frac{1}{2}(e^x + e^{-x})]^2 = \frac{1}{4}(e^{2x} + 2 + e^{-2x})$ $\sinh^2 x = [\frac{1}{2}(e^x - e^{-x})]^2 = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$ $\cosh^2 x - \sinh^2 x = \frac{1}{4}(2+2) = 1$	B1 B1 B1 ag **3**	For completion
	OR $\cosh x + \sinh x = \frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) = e^x$ B1 $\cosh x - \sinh x = \frac{1}{2}(e^x + e^{-x}) - \frac{1}{2}(e^x - e^{-x}) = e^{-x}$ B1 $\cosh^2 x - \sinh^2 x = e^x \times e^{-x} = 1$ B1		Completion
(ii)	$4(1+\sinh^2 x) + 9\sinh x = 13$ $4\sinh^2 x + 9\sinh x - 9 = 0$ $\sinh x = \frac{3}{4},\ -3$ $x = \ln 2,\ \ln(-3+\sqrt{10})$	M1 M1 A1A1 A1A1 ft **6**	(M0 for $1-\sinh^2 x$) Obtaining a value for $\sinh x$ Exact logarithmic form *Dep on M1M1* Max A1 if any extra values given
	OR $2e^{4x} + 9e^{3x} - 22e^{2x} - 9e^x + 2 = 0$ $(2e^{2x} - 3e^x - 2)(e^{2x} + 6e^x - 1) = 0$ M1 M1 $e^x = 2,\ -3+\sqrt{10}$ A1A1 $x = \ln 2,\ \ln(-3+\sqrt{10})$ A1A1 ft		Quadratic and / or linear factors Obtaining a value for e^x Ignore extra values *Dependent on M1M1* Max A1 if any extra values given *Just* $x = \ln 2$ *earns M0M1A1A0A0A0*
(iii)	$\frac{dy}{dx} = 8\cosh x \sinh x + 9\cosh x$ $= \cosh x(8\sinh x + 9)$ $= 0$ only when $\sinh x = -\frac{9}{8}$ $\cosh^2 x = 1 + (-\frac{9}{8})^2 = \frac{145}{64}$ $y = 4\times\frac{145}{64} + 9\times(-\frac{9}{8}) = -\frac{17}{16}$	B1 B1 M1 A1 **4**	Any correct form *or* $y = (2\sinh x + \frac{9}{4})^2 + \ldots\ (-\frac{17}{16})$ Correctly showing there is only one solution Exact evaluation of y or $\cosh^2 x$ or $\cosh 2x$ Give B2 (replacing M1A1) for -1.06 or better
(iv)	$\int_0^{\ln 2} (2 + 2\cosh 2x + 9\sinh x)\,dx$ $= [\ 2x + \sinh 2x + 9\cosh x\ ]_0^{\ln 2}$ $= \left\{ 2\ln 2 + \frac{1}{2}\left(4 - \frac{1}{4}\right) + \frac{9}{2}\left(2 + \frac{1}{2}\right)\right\} - 9$ $= 2\ln 2 + \frac{33}{8}$	M1 A2 M1 A1 ag **5**	Expressing in integrable form Give A1 for two terms correct $\sinh(2\ln 2) = \frac{1}{2}(4 - \frac{1}{4})$ *Must see both terms for M1* *Must also see* $\cosh(\ln 2) = \frac{1}{2}(2+\frac{1}{2})$ *for A1*

	OR $\int_0^{\ln 2} (e^{2x}+2+e^{-2x}+\frac{9}{2}(e^x-e^{-x}))\,dx$ M1 $=\left[\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}+\frac{9}{2}e^x+\frac{9}{2}e^{-x}\right]_0^{\ln 2}$ A2 $=\left(2+2\ln 2-\frac{1}{8}+9+\frac{9}{4}\right)-\left(\frac{1}{2}-\frac{1}{2}+\frac{9}{2}+\frac{9}{2}\right)$ M1 $=2\ln 2+\frac{33}{8}$ A1 ag		Expanded exponential form (M0 if the 2 is omitted) Give A1 for three terms correct $e^{2\ln 2}=4$ and $e^{-2\ln 2}=\frac{1}{4}$ *both seen* *Must also see* $e^{\ln 2}=2$ and $e^{-\ln 2}=\frac{1}{2}$ *for A1*
5 (i)	$\lambda=0.5$ $\lambda=3$ $\lambda=5$ 	B1B1B1 **3**	
(ii)	Ellipse	B1 **1**	
(iii)	$y=\sqrt{2}\cos(\theta-\frac{1}{4}\pi)$ Maximum $y=\sqrt{2}$ when $\theta=\frac{1}{4}\pi$	M1 A1 ag **2**	or $\sqrt{2}\sin(\theta+\frac{1}{4}\pi)$
	OR $\frac{dy}{d\theta}=-\sin\theta+\cos\theta=0$ when $\theta=\frac{1}{4}\pi$ M1 $y=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$ A1		
(iv)	$x^2+y^2=\lambda^2\cos^2\theta-2\cos\theta\sin\theta+\frac{1}{\lambda^2}\sin^2\theta$ $+\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta$ $=(\lambda^2+1)(1-\sin^2\theta)+(\frac{1}{\lambda^2}+1)\sin^2\theta$ $=1+\lambda^2+(\frac{1}{\lambda^2}-\lambda^2)\sin^2\theta$ When $\sin^2\theta=0$, $x^2+y^2=1+\lambda^2$ When $\sin^2\theta=1$, $x^2+y^2=1+\frac{1}{\lambda^2}$ Since $0\le\sin^2\theta\le 1$, distance from O, $\sqrt{x^2+y^2}$, is between $\sqrt{1+\frac{1}{\lambda^2}}$ and $\sqrt{1+\lambda^2}$	M1 M1 A1 ag M1 M1 A1 ag **6**	Using $\cos^2\theta=1-\sin^2\theta$
(v)	When $\lambda=1$, $x^2+y^2=2$ Curve is a circle (centre O) with radius $\sqrt{2}$	M1 A1 **2**	

(vi)		B4	**4**	A, E at maximum distance from O C, G at minimum distance from O B, F are stationary points D, H are on the *x*-axis Give ½ mark for each point, then round down Special properties must be clear from diagram, or stated *Max 3 if curve is not the correct shape*

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General Comments

Most candidates for this paper appeared to be well prepared across the range of syllabus topics, although a significant number were unfamiliar with converting from cartesian to polar coordinates. The marks were higher than last year, with about a quarter of the candidates scoring 60 marks or more (out of 72), and about 10% scoring fewer than 30 marks. The presentation of the candidates' work was generally very good, and most candidates appeared to have sufficient time to complete the paper. Some wasted time by using inefficient methods, or by deriving results which could be found in the formula book, such as the series for $\ln(1+x)$ and the logarithmic form of $\operatorname{arsinh} x$. Again, very many candidates lost marks for not showing sufficient working when the answer was given on the question paper. In Section B, the overwhelming majority of candidates chose the hyperbolic functions option.

Comments on Individual Questions

1) *(Polar coordinates, integration and Maclaurin series)*
The first and last parts of this question posed significant difficulties for a large number of candidates. The average mark for the question was about 11 (out of 18).

(a)(i) Many candidates found this to be an unconventional and surprising beginning, with even high-achieving candidates omitting this part; but there were many fully correct answers, some of which started with the polar equation and worked backwards. The most common error was to start with $x = \cos\theta$, $y = \sin\theta$.

(a)(ii) There were many fully correct graphs. The most common errors were to draw loops in the second and third quadrants, or to begin their loops too far away from the origin along the positive *x*-axis. Others exhibited cardioids and curves with many loops.

(b) This integration was efficiently done and was a good source of marks across the ability range, although there were some mistakes with the constants.

(c)(i) The Maclaurin series for $\ln(1+x)$ was usually produced correctly from the formula book, although some ignored the instruction to 'write down' the answer and obtained it by multiple differentiation. However, the series for $\ln(1-x)$ eluded many; candidates often did not realise that they were expected merely to replace *x* by $-x$ in the series already obtained, and began to differentiate (sometimes correctly), or just changed the signs randomly.

(c)(ii) The great majority of candidates subtracted here, although errors in part (i) prevented many from obtaining the correct answer, and a few tried dividing the two series. Some obtained the correct series by observing the logarithmic form of $\tanh x$ and the Maclaurin series for $\tanh x$, both given in the formula book; but this was not given full credit as the question required a derivation from part (i).

(c)(iii) This part was found to be difficult, and many candidates left it out. Those who made an attempt generally wrote out a few terms of the given series; slightly fewer observed that $\ln 3$ could be linked with the logarithmic expression when $x = ½$, and still fewer convincingly reconciled the two series. Many tried in vain to produce a convergent geometric series.

2) *(Complex numbers)*
The average mark on this question was about 12.

(i) The principles involved here were understood well, although many got off to a bad start by thinking that the modulus of $\sqrt{32}(1+\mathrm{j})$ was $\sqrt{32}$.

(ii) Most candidates realised that they could use the modulus and argument they had found in part (i).

(iii) Again, this was very well done and the method understood well. A few forgot to divide the argument by 3.

(iv) This part proved to be a considerable challenge, and few candidates achieved full marks, although the mark for k_1 was scored reasonably regularly. Candidates who achieved correct matchings sometimes had the *k* on the wrong side and produced the reciprocal of the required value, or left out minus signs.

3) *(Matrices)*
This was the best answered question, with an average mark of about 14. Many candidates chose to start with this one.

(i) The method for finding the inverse of a (3×3) matrix was very well known and often carried out correctly. A few candidates set $k = 4$ at the start of the question and left out *k* altogether.

(ii) Again, most candidates were familiar with what was required here and full marks were common. Virtually all could produce the matrices **P** and **D**. Then most knew how to find **M** as $\mathbf{PDP}^{-1}$, although a few used $\mathbf{P}^{-1}\mathbf{DP}$. A few attempted to find **M** from $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$ and simultaneous equations, which took a great deal of time and was rarely successful.

(iii) Candidates were expected to 'write down' the characteristic equation in factorised form, as the eigenvalues were given, but very many went straight to $\det(\mathbf{M} - \lambda\mathbf{I}) = 0$, which wasted time, although here it was often successful. The Cayley-Hamilton theorem was very well known and many were able to produce an expression for $\mathbf{M}^4$ in terms of $\mathbf{M}^2$, **M** and **I**, although there were quite a few slips here.

4) *(Hyperbolic functions)*
The average mark on this question was about 11. Candidates who wrote everything in terms of exponentials at the earliest opportunity were considerably less successful than those who realised that part (i) could usefully be applied in the following parts.

(i) Most candidates approached this confidently and produced a convincing proof.

(ii) For many candidates this part provided 6 quick and easy marks: obtaining and solving a quadratic equation for $\sinh x$, then using the logarithmic formula for arsinh (from the formula book). Some wasted time solving $\sinh x = \frac{3}{4}$ and $\sinh x = -3$ by forming quadratic equations in exponentials. Candidates who ignored the hint given by part (i), and began by converting the original equation to exponential form, produced a quartic equation which they could not solve.

(iii) Convincing explanations that $\mathrm{d}y / \mathrm{d}x = 0$ has only one solution were quite rare, and a surprisingly common error was differentiating $9 \sinh x$ to give $9 \sinh x$. Many candidates found the value of *x*, although this was not required; using part (i) the value of *y* can easily be found from the value of $\sinh x$.

(iv) Although many candidates integrated the expression efficiently and correctly, the great majority lost the last two marks through failure to show sufficient working, particularly when evaluating $\sinh(2 \ln 2)$. This working must be seen for the marks to be awarded when the answer is given.

5) *(Investigation of curves)*
Very few candidates (less than 1%) chose this question, and most of the attempts were fragmentary.